How to Award Pro-competitive Forward Contracts

The case of electricity auctions

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Motivation

Market power concerns in electricity markets have led regulators to impose **forward contract obligations**:

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- [Several other vertical commitments, including **vertical integration**]

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3. How should contracts be **awarded** for them to be **pro-competitive**?

Related literature

Literature on (endogenous) forward contracts:

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Despite discreteness, **we recover under-pricing because we relax the flat common value assumption**

Overview and main findings

- Multi-unit **uniform-price auction model** with complete information:
  - Supply functions with a **finite number of steps**
  - Firms own a portfolio of technologies (**increasing marginal costs**)
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- **Conclusion**:
  - Award contracts so as to (virtually) reduce firms' asymmetries

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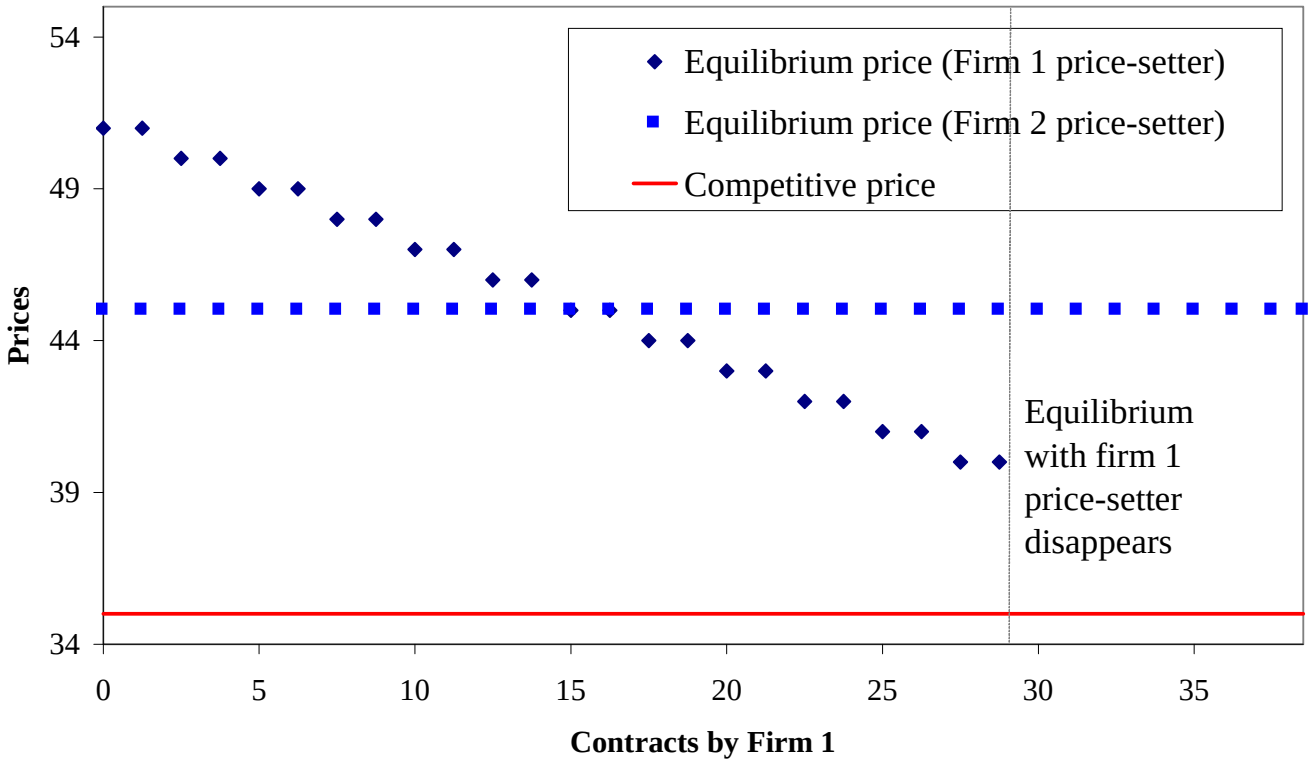

Figure 1: Forward contracts by large firm

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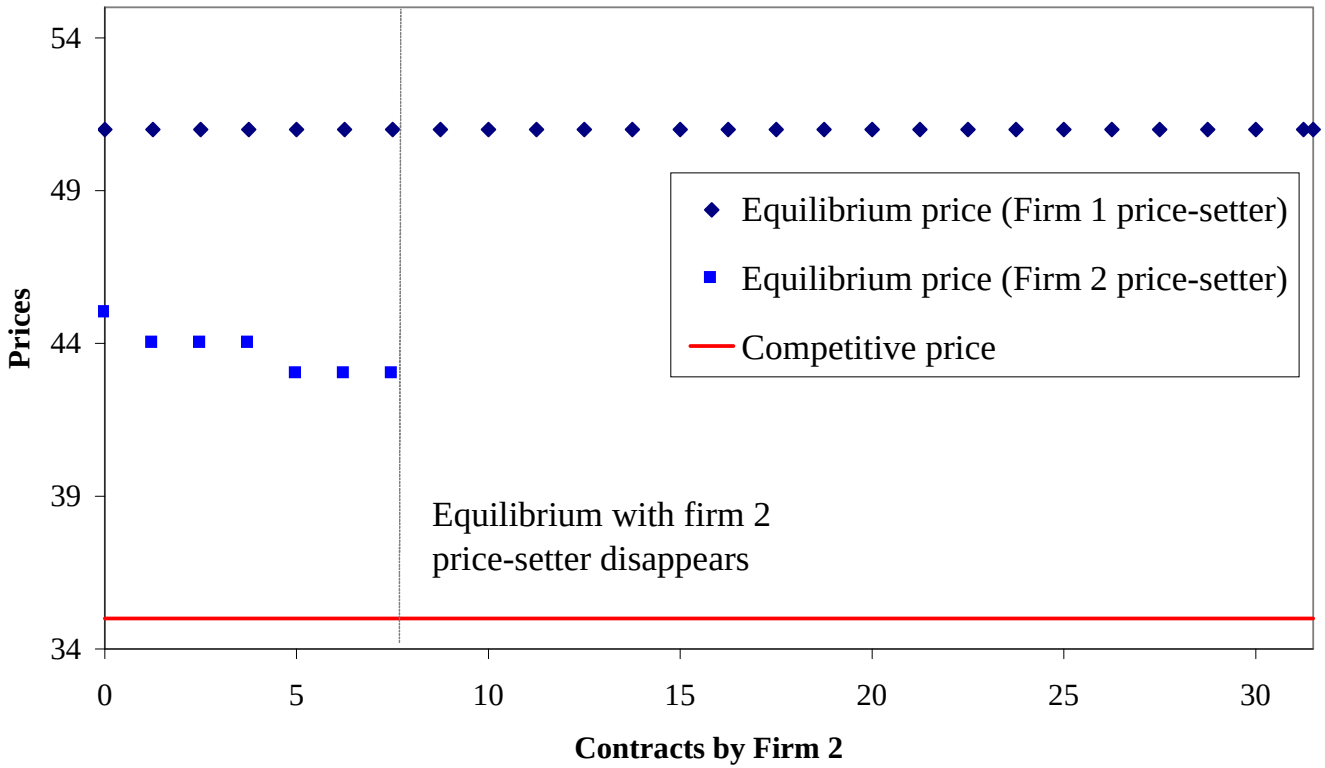

Figure 2: Forward contracts by small firm

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- **Marginal cost functions** are non-decreasing **step functions**

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- Firms submit a **finite** number of **price-quantity** pairs
- **Bid functions**: non-decreasing left continuous **step functions**

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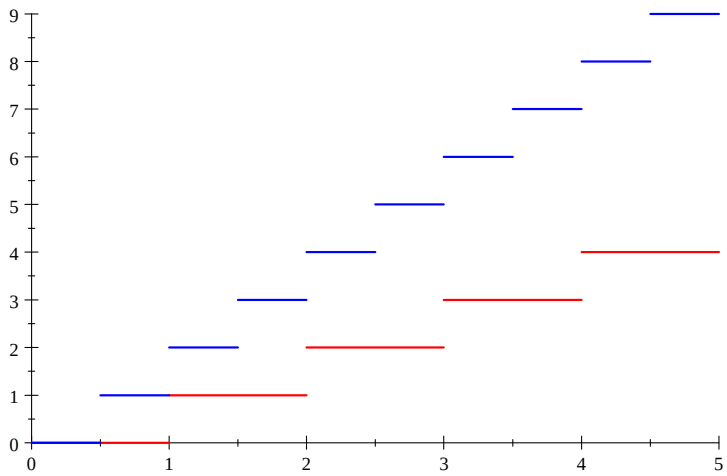

Figure 3: Example with marginal costs $c_i = \{(0,1),(1,2),...\}$ (red) and bid function $b_i = \{(0,0.5),(1,1),...\}$ (blue)

Dispatch and pricing

Dispatch:

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- The **stop-out price**, p^*, is equal to the price of the last accepted bid
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Pricing:

- All dispatched output receives p^* (**uniform-price auction**)

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- Firm i's **chooses a bid function** that max. π_i given rivals' bids

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- Wlog, we omit contract revenues, $\tau_i = 0$

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  - $p^* = 2$ and $q_1 = 1$
  - $\pi_1 = 2 > \pi_1^c = 1$, i.e., the deviation is profitable!

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- Given $p^* = 2$, firm 2 cannot profitably change its **output** (as a price-taker, it's producing all it can w/o incurring in losses)

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Contracts eliminate one equilibrium
but the equilibrium **price remains the same**

Asymmetric firms

- Assume now that firms are **asymmetric**:
  - Firm 1 has 4 units, with $c_1 = \{0, 1, 2, 2\}$ [*large firm*]
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No-contracts case: multiplicity of equilibrium outcomes

1. Firm 2 bids at MC; firm 1 sets $p^* = P = 3$ [*high-price equilibrium*]
2. Firm 1 bids at MC; firm 2 sets $p^* = 2$ [*low-price equilibrium*]

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Forward contracts are **pro-competitive**

They (weakly) decrease prices

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Contracts for the small firm

Suppose $x_1 = 0$ and $x_2 \in (1, 2)$

The low-price equilibrium disappears:

- If firm 1 bids at MC, firm 2 responds by also bidding at MC
- ...but if firm 2 bids at MC, firm 1 responds by setting $p^* = 3$

The high-price equilibrium still exists

Forward contracts are **anti-competitive**

They (weakly) increase prices

The impact of forward contracts

	Equilibrium Prices	
	No contracts	$x_i \in (1,2] > x_j = 0$
Symmetric firms	$\{2,2\}$	$\{\varnothing,2\}$

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- The **symmetry assumption** would hide these effects

Analysis of the Model

Weakly-dominated strategies

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Weak-dominance arguments eliminate...

1. below marginal cost bidding for $q_i > x_i$ (net-seller).
2. above marginal cost bidding for $q_i < x_i$ (net-buyer).

Equilibrium bidding by the non-price-setters

Proposition

At any Nash equilibrium in which firm i is a price-setter, all other firms produce the same output as if they bid at marginal costs.

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Corollary

At any non-competitive Nash equilibrium, there is a unique price-setter.

Equilibrium bidding by NPS and PS

- NPS behave as price-takers (need not bid at MC)

$$q_j^{NPS}\left(p^*\right) \in \arg\max_{q_j^*} \pi_j^{NPS}\left(p^*; q_j^*\right),$$

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$$\pi_i^{PS}\left(p^*; b_{-i}\right) = p\left[q_i^*\left(p^*; b_{-i}\right) - x_i\right] - C\left(q_i^*\left(p^*; b_{-i}\right)\right) + \tau_i x_i.$$

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- **Note:** The profit function may fail to be differentiable.

Incentive's to raise the price

Suppose the PS raises the price from p^* to p' :

$$\Delta\pi_i^{PS} = \left[p' - p^*\right]\left[q_i^*\left(p'; b_{-i}\right) - x_i\right] - \int_{q_i^*(p';b_{-i})}^{q_i^*(p^*;b_{-i})} \left[p^* - c_i\left(z\right)\right] dz.$$

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Corollary

The PS' profit max. price is non-decreasing in its contracts.

Equilibrium characterization

Necessary and sufficient conditions for an equilibrium:

1. All firms must **optimize conditionally on** their **identities:**
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- There might coexist multiple equilibria with different price-setters.
- But, for a given PS, all equilibria are outcome equivalent
  - regardless of whether NPS bid at MC or not.

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Contracts have the potential to **improve market performance** only if they **mitigate existing asymmetries**

Contracts to large firm (low asymmetry)

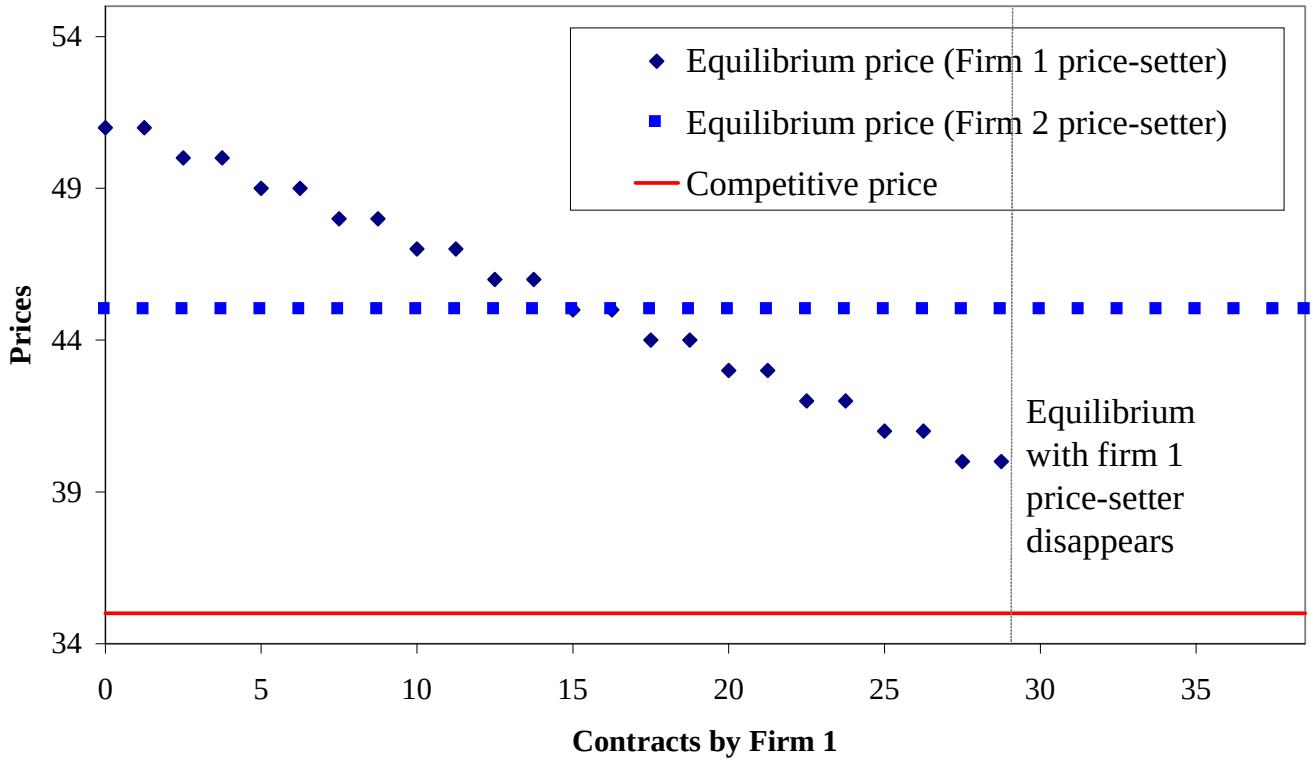

Figure 4: Contracts by large firm (capacity shares (55%,55%))

Contracts to large firm (high asymmetry)

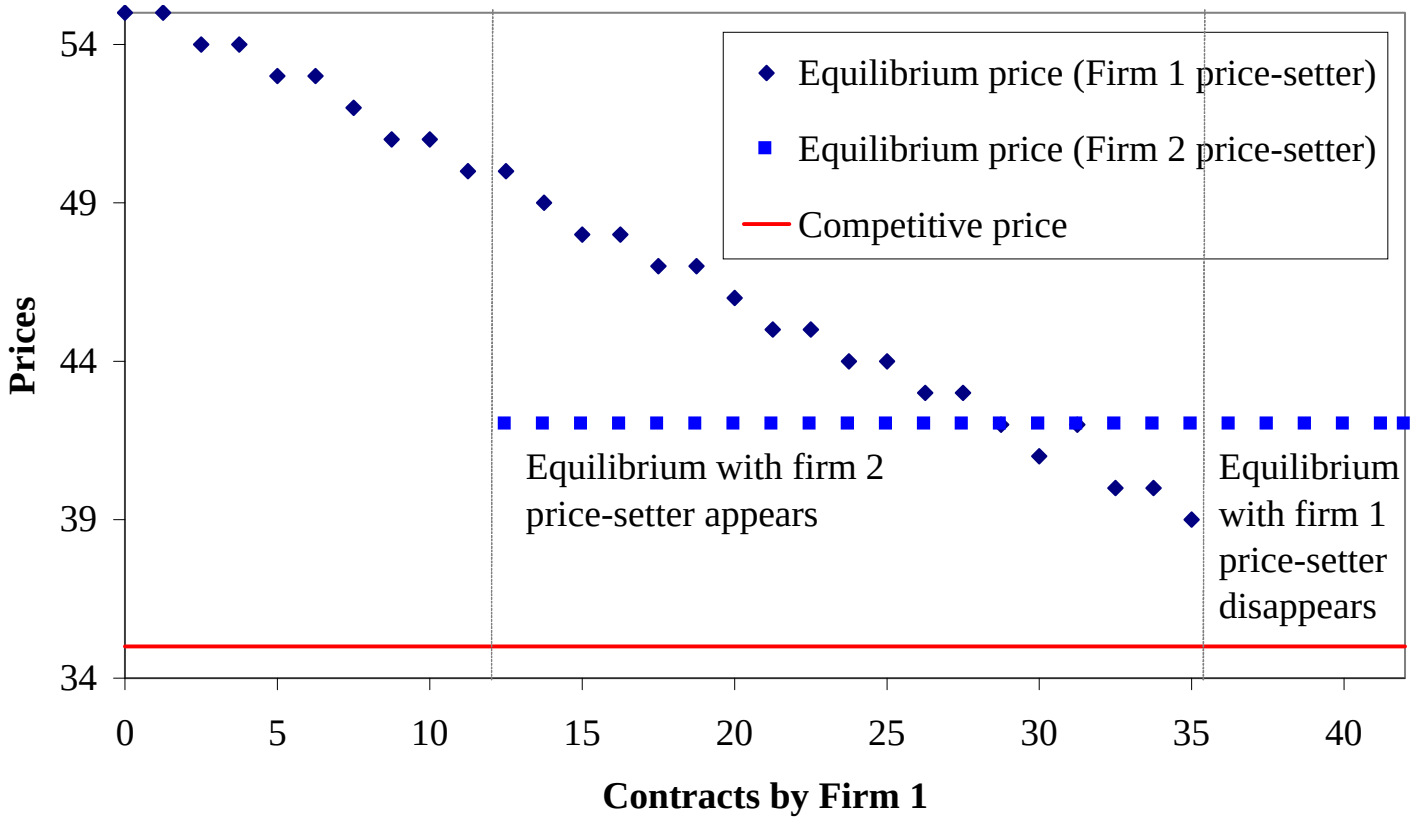

Figure 5: Contracts by large firm (capacity shares (60%,40%))

Contracts to small firm (low asymmetry)

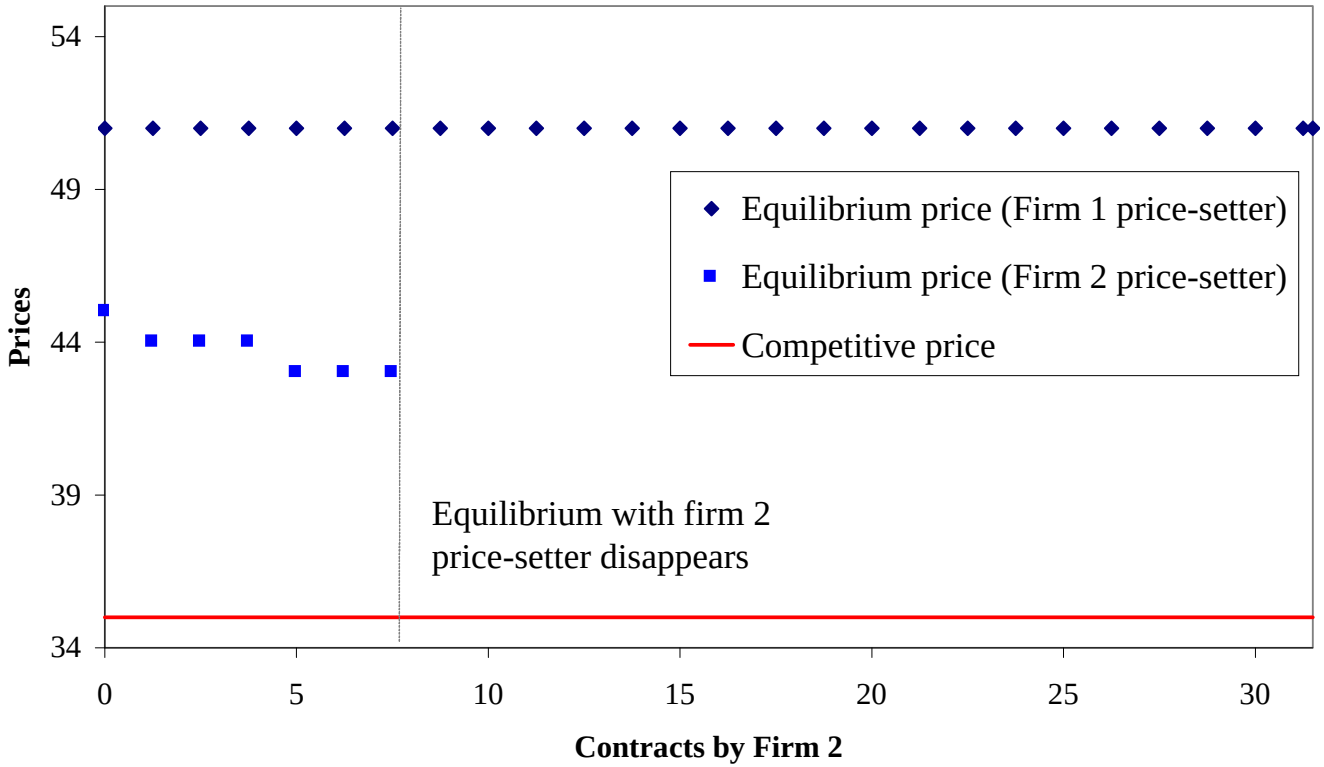

Figure 6: Contracts by small firm (capacity shares (55%,55%))

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2. Are contracts **pro-competitive** or **anti-competitive?**
   - The scope of contracts to improve market performance depends on:
     - **Market structure**
     - Total **contract volume** and its **distribution** across firms
   - Anti-competitive if they make net-positions more asymmetric.
3. **How should contracts be awarded?**
   - In order to (virtually) reduce asymmetries across firms:
     - Impose forward contract obligations on large firms.
     - Allow medium-sized competitors to buy such contracts.

Thank You.

Paper available at www.eco.uc3m.es/nfabra